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Signal for the Quark-Hadron Phase Transition in Rotating Hybrid Stars

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Abstract

For the past 20 years it had been thought that the coexistence phase of the confined hadronic and quark matter phases, assumed to be a first order transition, was strictly excluded from neutron stars. This, however, was due to a seemingly innocuous idealization which has approximated away important physics. The reason is that neutron stars constitute multi-component bodies rather than single-component ones formerly (and incorrectly) used to describe the deconfinement phase transition in neutron stars. So, contrary to earlier claims, 'neutron' stars may very well contain quark matter in their cores surrounded by a mixed-phase region of quark and hadronic matter. Such objects are called *hybrid* stars. The structure of such stars as well as an observable signature that could signal the existence of quark matter in their cores are discussed in this paper.

1 Introduction

In all earlier work of the last two decades on the quark–hadron phase transition in neutron stars, a degree of freedom was frozen out which yielded a description of the transition as a *constant* pressure one, as illustrated schematically in Fig. 1. Since pressure in a star is monotonically decreasing (because of hydrostatic equilibrium), this had the explicit consequence of excluding the coexistence phase of hadrons and quarks (region a–b in Fig. 1) from neutron stars. The degree of freedom that was frozen out is the possibility of reaching the lowest possible energy state by rearranging electric charge between the regions of hadronic matter and quark matter in phase equilibrium. Because of this freedom the pressure in the mixed phase *varies* as the proportions of the phases, and therefore the coexistence phase is not excluded from the star [1, 2, 3].

The physical reason behind this is the conservation of baryon charge and electric charge in neutron star matter. Correspondingly, there are two chemical potentials – one associated with baryon charge and the other associated with electric charge – and therefore the transition of baryon matter to quark matter is to be determined in three-space spanned by pressure and the chemical potentials of the electrons

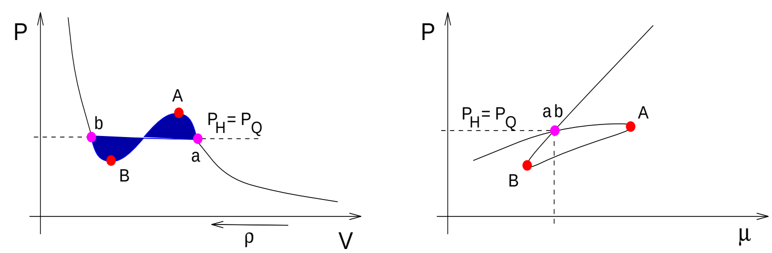

Figure 1: Phase transition in a 'simple' body (only one conserved entity) for which pressure stays constant in the quark–hadron transition region 'a–b'. Left: volume dependence of pressure for a given temperature. Points 'A' and 'B' denote metastable states. Right: same as left-hand side, but for the chemical potential as independent variable. The labels refer to the same points as in the figure to the left. Phase equilibrium 'a–b' is mapped onto the single point 'a b' where the curves intersect.

and neutrons (rather than two-space), as schematically illustrated in Fig. 2. This circumstance has not been realized in the numerous investigations published on this topic earlier [1].

2 Three neutron star models

To explore the implications of the mixed phase for the structure of neutron stars, we shall employ a collection of different models for the equation of state derived for

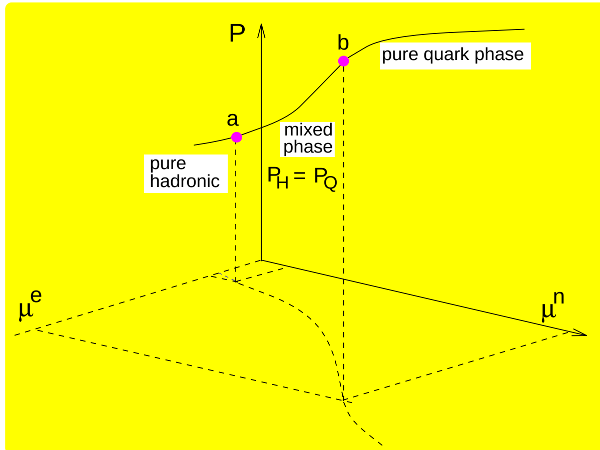

Figure 2: Phase transition in a body (in our case a neutron star) with *two* conserved entities, that is, electric charge and baryon charge. In contrast to the phase transition in a body with only one conserved charge, shown in Fig. 1, here pressure in the mixed phase varies with density. Therefore the mixed phase is not excluded from neutron stars, as is the case for the equation of state shown in Fig. 1.

three different assumptions about the composition of 'neutron' star matter.

2.1 Neutron stars

In the most primitive conception, a neutron star is constituted from neutrons. At a slightly more accurate representation, a beta stable compact star will contain neutrons and a small number of protons whose charge is balanced by leptons. We represent the interactions among baryons in the relativistic mean field theory. Details can be found elsewhere [4, 5, 6]. The coupling constants in the theory are chosen so that for symmetric nuclear matter, the five important bulk properties (energy per baryon, incompressibility, effective nucleon mass, asymmetry energy, saturation density) are reproduced [7, 8].

2.2 Hyperon stars

At the densities in the interior of neutron stars, the neutron chemical potential will exceed the mass (modified by interactions) of various members of the baryon octet [9]. So in addition to neutrons, protons and electrons, neutron stars are expected to have populations of hyperons which together with nucleons and leptons are in a charge neutral equilibrium state. Interactions among the baryons are incorporated, and coupling constants chosen, as above [4, 5, 6, 7]. Hyperon coupling constants are chosen: (1) to reproduce the binding of the lambda in nuclear matter, (2) to be

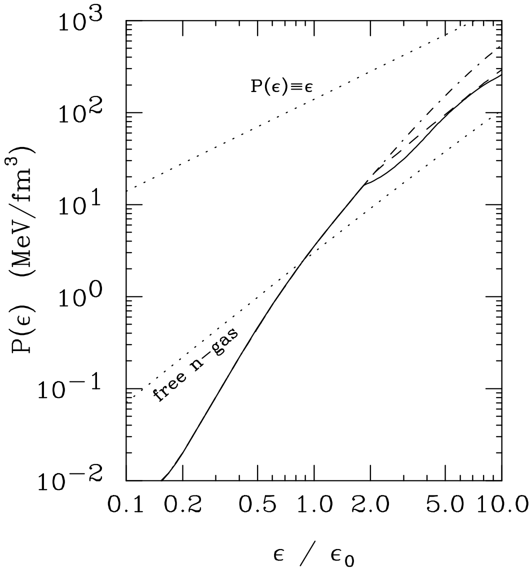

Figure 3: Three models for the equation of state. Solid curve: equation of state of hybrid star ($G_{\mathrm{B180}}^{\mathrm{K240}}$), dashed curve: hyperon star ($G_{\mathrm{M78}}^{\mathrm{K240}}$), dash-dotted curve: neutron star ($G_{\mathrm{M78}}^{\mathrm{K240}}$), protons and neutrons only). (For details, see Ref. [8].)

compatible with hypernuclei, and (3) to support at a minimum a neutron star of at least $1.5\,M_\odot$ [1].

2.3 Hybrid stars

How to handle phase equilibrium in dense neutron star matter having two conserved charges, baryon and electric, is described in detail elsewhere [1]. As we have seen qualitatively in Section 1, the properties of a phase transition in a multi-component system is very different from the familiar one of a single-component system formerly (and incorrectly) used to describe the deconfinement phase transition in neutron stars [1, 2]. Models for the equation of state of neutron, hyperon, and hybrid star matter are shown in Fig. 3. One sees that the transition of confined baryonic matter to quark matter sets in at about twice nuclear matter density ϵ_0 ($= 140$ MeV/fm^3), which leads to an additional softening of the equation of state. Pure quark matter is obtained for densities $\gtrsim 7\epsilon_0$.

3 Sequences of rotating stars with constant baryon number

Neutron stars are objects of highly compressed matter so that the geometry of space-time is changed considerably from flat space-time. Thus for the construction of realistic models of rapidly rotating pulsars one has to resort to Einstein's theory of general relativity. In the case of a star rotating at its *absolute* limiting rotational period, that is, the Kepler (or mass-shedding) frequency, Einstein's field equations,

$$\mathcal{R}^{\kappa\lambda} - \frac{1}{2}\, g^{\kappa\lambda}\, \mathcal{R} = 8\,\pi\, \mathcal{T}^{\kappa\lambda}(\epsilon, P(\epsilon))\,, \tag{1}$$

are to be solved selfconsistently in combination with the general relativistic expression which describes the onset of mass-shedding at the equator [6, 10, 11]:

$$\Omega_{\mathrm{K}} = \omega + \frac{\omega'}{2\psi'} + e^{\nu-\psi}\sqrt{\frac{\nu'}{\psi'} + \Big(\frac{\omega'}{2\psi'}e^{\psi-\nu}\Big)^2}\,. \tag{2}$$

The metric for a rotating star, suitable for both the interior and exterior, reads [10, 12, 13],

$$ds^2 = -\,e^{2\nu}dt^2 + e^{2\lambda}dr^2 + e^{2\mu}d\theta^2 + e^{2\psi}[d\phi - \omega\, dt]^2\,. \tag{3}$$

Because of the underlying symmetries, the metric functions ν, ψ, μ, and λ are independent of t and ϕ but depend on r, θ and Ω. The ω denotes the angular velocity of the local inertial frames (frame dragging frequency) and depends on the same variables as the metric. The frequency $\bar\omega \equiv \Omega - \omega(r,\theta,\Omega)$, is the star's rotational frequency relative to the frequency of the local inertial frames, and is the one on which the centrifugal force acting on the mass elements of the rotating star's fluid depends [14]. The quantities $\mathcal{R}^{\kappa\lambda}$, $g^{\kappa\lambda}$, and $\mathcal{R}$ denote respectively the Ricci tensor, metric tensor, and Ricci scalar (scalar curvature). The dependence of the energy-momentum tensor $\mathcal{T}^{\kappa\lambda}$ on pressure and energy density, P and ϵ respectively, is indicated in Eq. (1). The primes in (2) denote derivatives with respect to Schwarzschild radial coordinate, and all functions on the right are evaluated at the star's equator. All the quantities on the right-hand side of (2) depend also on Ω_{K}, so that it is not an equation for Ω_{K}, but a transcendental relationship which the solution of the equations of stellar structure, resulting from Eq. (1), must satisfy if the star is rotating at its Kepler frequency. (Details can be found in [6].)

The outcome of two selfconsistent calculations, one for a hybrid and the other for a conventional hyperon star, is compared in Figs. 4 and 5. The stars' baryon number is kept constant during spin-down from the Kepler frequency to zero rotation, as it should be. The frequency Ω is assumed to be constant throughout the star's fluid since uniform rotation is the configuration that minimizes the mass-energy at specified baryon number and angular momentum [15].

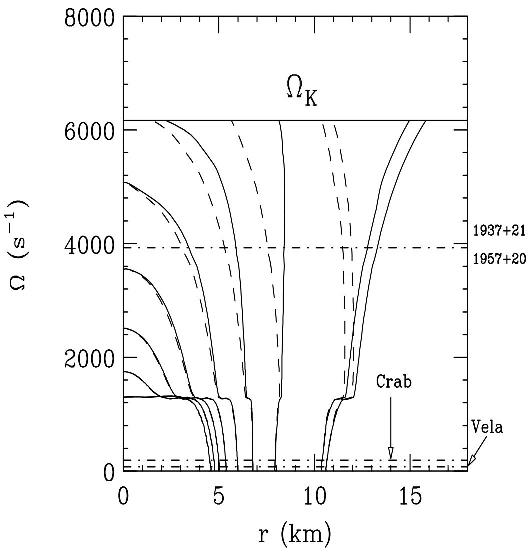

Figure 4: Frequency dependence of quark structure in rotating hybrid stars. The radial direction is along the star's equator (solid curves) and pole (dashed curves). The nonrotating star mass is $\sim 1.42\,M_\odot$. The rotational frequency ranges from zero to Kepler.

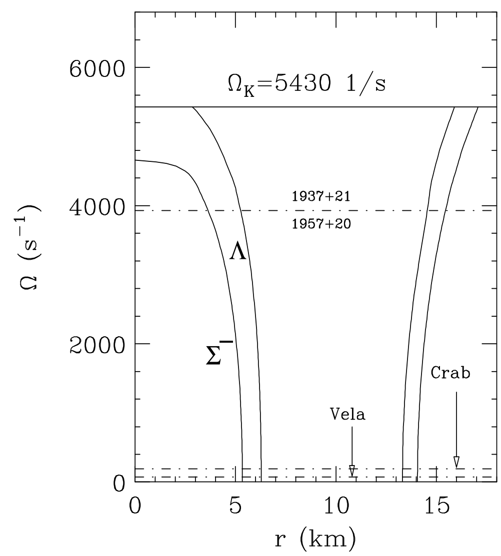

Figure 5: Same as Fig. 4, but for a conventional hyperon star ($M \sim 1.40\,M_\odot$), that is, transition to quark matter is suppressed. The Σ^- is absent for $\Omega \gtrsim 4700$ s^{-1} because the central density falls below the threshold density of the Σ^- particle.

According to the mass of a hybrid star, it may consist of an inner sphere of purely quark matter (lower-left portion of Fig. 4 for which $\Omega \lesssim 1250$ s^{-1} and $r \lesssim 4.5$ km), surrounded by a few kilometers thick shell of mixed phase of hadronic and quark matter arranged in a lattice structure, and this surrounded by a thin shell of hadronic liquid, itself with a thin crust of heavy ions [2]. The Coulomb lattice structure of varying geometry introduced to the interior of neutron stars [2], which may have dramatic effects on pulsar observables including transport properties and the theory of glitches, is a consequence of the competition of the Coulomb and surface energies of the hadronic and quark matter phase. This competition establishes the shapes, sizes and spacings of the rarer phase in the background of the other (that is, for decreasing density: hadronic drops, hadronic rods, hadronic plates immersed in quark matter followed by quark plates, quark rods and quark drops immersed in hadronic matter) so as to minimize the lattice energy. For an investigation of the structure of the mixed phase of baryons and quarks predicted by Glendenning, we refer to [16].

As a rotating hybrid star spins down it becomes less deformed and the central density *rises*. For some pulsars the mass and initial rotational frequency Ω may be such that the central density rises from below the critical density for dissolution of

baryons into their quark constituents. This is accompanied by a sudden shrinkage of the hybrid star, which dramatically effects its moment of inertia and hence the braking index of a pulsar, as we shall see in the next sections.

4 Moment of inertia

Elsewhere we have obtained an expression for the moment of inertia of a relativistic star that accounts for the centrifugal flattening of the star [11, 17]. It is given by

$$I(\Omega) = 4\pi \int_0^{\pi/2} d\theta \int_0^{R(\theta)} dr \, \frac{e^{\lambda+\mu+\nu+\psi} \, [\epsilon + p]}{e^{2(\nu-\psi)} - \bar{\omega}^2} \, \frac{\Omega - \omega}{\Omega} \, . \tag{4}$$

The radial distribution of energy density and pressure, $\epsilon(r)$ and $p(r)$, are found from the solution of the equations for rotating relativistic stars. For slow rotation this expression reduces to the well known, and frequency *independent* result [17].

We show how the moment of inertia changes with frequency in Fig. 6 for stars having the same baryon number but different constitutions, as described in Sect. 2. The two curves without a drop at low frequencies are for conventional neutron (dot-dashed line) and hyperon (dashed) stars of non-rotating mass $M \sim 1.45\, M_{\odot}$. The solid line is for a hybrid star of roughly the same mass and baryon number. The shrinkage of the hybrid star due to the development of a quark matter core at low frequencies, known from Fig. 4, manifests itself in a sudden reduction of I, which is the more pronounced the bigger the quark matter core (i.e., the smaller Ω) in the center of the star.

5 Evolution of braking index of pulsars

Pulsars are identified by their periodic signal believed to be due to a strong magnetic field fixed in the star and oriented at an angle from the rotation axis. The period of the signal is therefore that of the rotation of the star. The angular velocity of rotation decreases slowly but measurably over time, and usually the first and occasionally second time derivative can also be measured. Various energy loss mechanisms could be at play such as the dipole radiation, part of which is detected on each revolution, as well as other losses such as ejection of charged particles [18]. The measured frequency and its time derivative have been used to estimate the spin-down time or age of pulsars. The age is very useful for classifying and understanding pulsar phenomena such as glitch activity.

Let us assume, as usual, that the pulsar slow-down is governed by a single mechanism or several mechanisms having the same power law. Let us write the energy balance equation as

$$\frac{dE}{dt} = \frac{d}{dt} \left\{ \frac{1}{2} \, I \, \Omega^2 \right\} = - \, C \, \Omega^{n+1} \, , \tag{5}$$

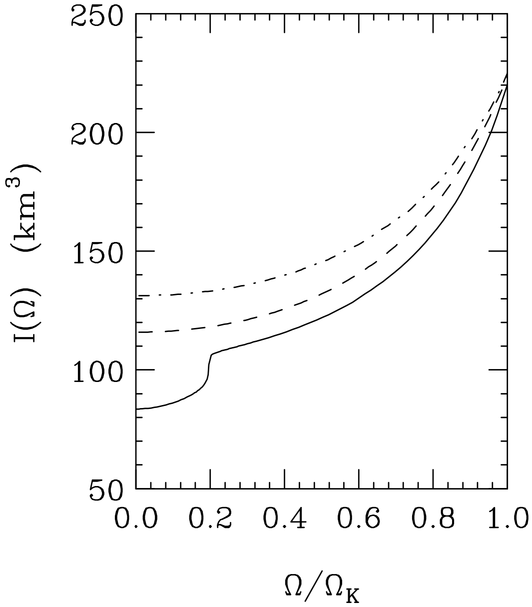

Figure 6: Moment of inertia as a function of rotational frequency (in units of Kepler) for three neutron stars (solid curve: hybrid star, dashed curve: hyperon star, dash-dotted curve: neutron star made up of only protons and neutrons) with different constitutions as described in Sect. 2. The baryon number is constant along each curve. The development of a quark matter core for decreasing frequency (increasing density) causes a sudden reduction of I.

where, for magnetic dipole radiation, $C = \frac{2}{3} m^2 \sin^2 \alpha$, $n = 3$, m is the magnetic dipole moment and α is the angle of inclination between magnetic moment and rotation axis. If, as is customary, the angular velocity Ω is regarded as the only time-dependent quantity, one obtains the usual formula for the rate of change of pulsar frequency,

$$\dot{\Omega} = -\, K\, \Omega^n \,, \tag{6}$$

with K a constant and n, the *braking index*. From the braking law (6) one usually defines from its solution, the spin-down age of the pulsar

$$\tau = -\, \frac{1}{n-1} \frac{\Omega}{\dot{\Omega}} \,. \tag{7}$$

However the moment of inertia is *not* constant in time but responds to changes in rotational frequency, as can be seen in Fig. 6, more or less according to the softness or stiffness of the equation of state (that is, the star's internal constitution) and according as the stellar mass is small or large. This response changes the value of the braking index in a frequency dependent manner, even if the sole energy-loss mechanism were *pure* dipole as in Eq. (5). Thus during any epoch of observation, the

braking index will be measured to be different from $n = 3$ by a certain amount. How much less depends, for any given pulsar, on its rotational frequency and for different pulsars of the same frequency, on their mass and on their internal constitution.

When the frequency response of the moment of inertia is taken into account, Eq. (6) is replaced by

$$\dot{\Omega} = -\,2IK\,\frac{\Omega^n}{2I + I'\Omega} = -\,K\,\Omega^n \Big\{ 1 - \frac{I'}{2I}\,\Omega + \Big(\frac{I'}{2I}\,\Omega\Big)^2 - \cdots \Big\}\,, \tag{8}$$

where $I' \equiv dI/d\Omega$ and $K = C/I$. This explicitly shows that the frequency dependence of $\dot{\Omega}$ corresponding to *any* mechanism that absorbs (or deposits) rotational

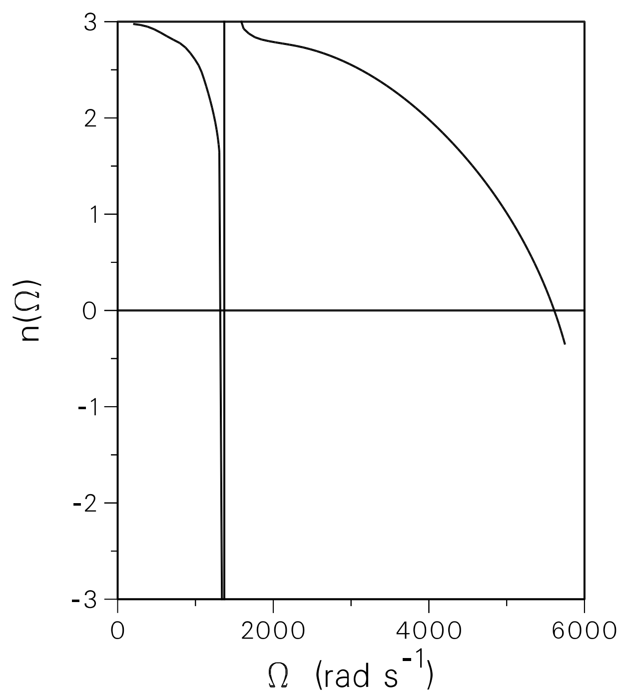

Figure 7: Braking index as a function of rotational frequency for a hybrid star. The dip at low frequencies is driven by the phase transition of baryonic matter into quark matter. The overall reduction of n below 3 is due to the frequency dependence of I and therefore holds for all three types of stars (neutron, hyperon, and hybrid).

energy such as Eq. (5) cannot be a power law, as in Eq. (6) with K a constant. It must depend on the mass and internal constitution of the star through the response of the moment of inertia to rotation as in Eq. (8).

Equation (8) can be represented in the form of Eq. (6) (but now with a frequency dependent prefactor) by evaluating

$$n(\Omega) = \frac{\Omega\,\ddot{\Omega}}{\dot{\Omega}^2} = n - \frac{3\,I'\,\Omega + I''\,\Omega^2}{2\,I + I'\,\Omega}\,. \tag{9}$$

Therefore the effective braking index depends explicitly and implicitly on Ω. The right side reduces to a constant n only if I is independent of frequency. But his cannot be, not even for slow pulsars if they contain a quark matter core. The centrifugal force ensures the response of I to Ω. As an example, we show in Fig. 7 the variation of the braking index with frequency for the rotating hybrid star of Fig. 4. For illustration we assume dipole radiation. As before, the baryon number of the star is kept constant. Because of the structure in the moment of inertia, driven by the phase transition into the deconfined quark matter phase, the braking index deviates dramatically from 3 at small rotation frequencies. Such an anomaly in $n(\Omega)$ is not obtained for conventional neutron or hyperon stars because their moments of inertia increase smoothly with Ω (cf. Fig. 6). The observation of such an anomaly in the timing structure of pulsars may thus be interpreted as a signal for the development of quark-matter cores in the centers of pulsars. As shown in [19], the duration over which the braking index would be anomalous may be 1/100'th of the active pulsar lifetime. Given that $\sim 10^3$ pulsars are known (actually > 700 as of this date), about 10 of these may be signaling the phase transition!

6 Summary

Whether or not the cores of neutron stars are in the deconfined quark matter phase makes little difference to their static properties such as the range of possible masses, sizes, or even their limiting rotational frequencies. However we find that dramatic effects may occur in the timing structure of a pulsar's spin–down, that is, in the so-called braking index of a pulsar: the possible phase transition of confined baryonic matter into its deconfined phase may register itself in a dramatic change of the braking index, which is completely absent for stars entirely made up of confined baryonic matter. We estimate that about 10 out of the presently known ~ 700 pulsars could be signaling the phase transition.

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